



# Generalized Finite-Length Fibonacci Sequences in Healthy and Pathological Human Walking: Comprehensively Assessing Recursivity, Asymmetry, Consistency, Self-Similarity, and Variability of Gaits

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Healthy and pathological human walking are here interpreted, from a temporal point of view, by means of dynamics-on-graph concepts and generalized finite-length Fibonacci sequences. Such sequences, in their most general definition, concern two sets of eight specific time intervals for the newly defined *composite gait cycle*, which involves two specific couples of overlapping (left and right) gait cycles. The role of the golden ratio, whose occurrence has been experimentally found in the recent literature, is accordingly characterized, without resorting to complex tools from linear algebra. Gait recursivity, self-similarity, and asymmetry (including double support sub-phase consistency) are comprehensively captured. A new gait index, named  $\Phi$ -*bonacci gait number*, and a new related experimental conjecture—concerning the position of the foot relative to the tibia—are concurrently proposed. Experimental results on healthy or pathological gaits support the theoretical derivations.

**Keywords:** gait analysis, walking gait, asymmetry, self-similarity, golden ratio, fibonacci sequence, locomotion, neuroscience

## 1. INTRODUCTION

Four time intervals—associated with the durations of gait cycle, swing, stance and double support phases—characterize, from a temporal point of view, symmetric and recursive human walking (Dugan and Bat, 2005). Recently, the ratio between swing and double support phases durations has been experimentally recognized in Iosa et al. (2013)<sup>1</sup> to be close, in healthy subjects symmetrically and recursively walking at comfortable speed of about 4 km/h (Cavagna and Margaria, 1966), to the golden ratio  $\phi = (1 + \sqrt{5})/2 \approx 1.618$ . Such an irrational number  $\phi$  is the positive solution to the equation  $x^2 = 1 + x$ . It is related to the Euclid's problem of cutting in a self-proportional way a

<sup>1</sup>Spatio-temporal gait parameters are analyzed in Iosa et al. (2013) by using a stereo-photogrammetric system with 6 cameras.